Hypothesis Testing for Means

Lecture 32
Sections 10.1-10.2

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Outline

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Introduction

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  - Is a given hypothesis concerning μ true?
  - What is the value of μ?
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- In one way, it will be very different.

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6. State the decision.

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6. State the decision.
7. State the conclusion.

An Example

- Has the price of gasoline gone up?
- It was \$3.69 a few days ago.
- In an attempt to determine whether the price for a gallon of regular gasoline is greater than \$3.69, a reporter samples 36 service stations.
- He finds an average price of \$3.75.
- The population standard deviation σ is (somehow) known to be \$0.12.
- At the 5% level of significance, test the hypothesis that the average price of a gallon of gasoline is greater than \$3.69.

Outline

The Hypotheses

- Tell what μ represents.
- The null hypothesis gives a hypothetical value μ_0 for the population mean.

$$H_0 : \mu = \mu_0.$$

- The alternative hypothesis contradicts H_0 in one of three ways.
  - $H_1 : \mu < \mu_0$.
  - $H_1 : \mu > \mu_0$.
  - $H_1 : \mu \neq \mu_0$.

The Hypotheses

Example (Step 1)

(1) Let μ represent the average price of a gallon of regular gas.

$H_0 : \mu = 3.69$

$H_1 : \mu > 3.69$

Outline

The Level of Significance

- State the level of significance (the value of α).

The Level of Significance

Example (Step 2)

(2) Let $\alpha = 0.05$.

Outline

The Test Statistic

- The choice of test statistic is where things get a little complicated.
- The choice will depend on the sample size and what is known about the population. (Details tomorrow.)
- If we assume that σ is known and that either
  - The sample size n is at least 30, or
  - The population is normal,

  then the Central Limit Theorem for Means will apply, telling us that $\overline{x}$ is normal with mean μ_0 (if H_0 is true) and standard deviation $\frac{\sigma}{\sqrt{n}}$.

The Sampling Distribution of $\overline{x}$

- Therefore, the test statistic, for now, is

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}.$$

The Test Statistic

Example (Step 3)

(3) The test statistic is

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}.$$

Outline

The Value of the Test Statistic

- Substitute the values of $\overline{x}$, μ_0, σ, and n into the formula to get the value of z.

The Value of the Test Statistic

Example (Step 4)

(4)

- $\mu_0 = 3.69$.
- $\overline{x} = 3.75$.
- $\sigma = 0.12$.
- $n = 36$.
- Therefore, $z = \dfrac{3.75 - 3.69}{0.12/\sqrt{36}} = \dfrac{0.06}{0.02} = 3.000$.

Outline

The *p*-Value

- To find the *p*-value, use `normalcdf` on the TI-83.
- For one-tailed tests, compute either
  - *p*-value = `normalcdf(-E99,`*z*`)`, or
  - *p*-value = `normalcdf(`*z*`,E99)`.
- For a two-tailed test, compute the area of the appropriate tail, and then double it.

The *p*-Value

Example (Step 5)

(5) *p*-value $=$ `normalcdf(3,E99)` $=$ 0.001350.

Outline

The Decision

- Reject the null hypothesis if the *p*-value is less than α.
- Otherwise, accept it.
- Write either "Reject H_0" or "Accept H_0."

The Decision

Example (Step 6)

(6) Reject H_0.

Outline

The Conclusion

- State the conclusion in plain English, using the terminology of the original problem, not the statistical jargon.

The Conclusion

Example (Step 7)

(7) The average price of a gallon of regular gasoline is greater than \$3.69.

Outline

Exercise

Let's Do It! 10.2, page 620.

- Under normal circumstances, mice complete a maze in an average time of 18 seconds, with a standard deviation of 2 seconds.
- A researcher introduces loud noises. Will this cause the mice to run the maze faster?
- A sample of 10 mice has an average time of 17 seconds.
- Assume that their run times are normally distributed.
- The population standard deviation is known to be 2 seconds.
- Test the hypothesis at the 10% level that the average run time is less when there is loud noise.

Outline

Hypothesis Testing on the TI-83

TI-83 Hypothesis Testing for the Mean

- Press `STAT`.
- Select `TESTS`.
- Select `Z-Test`.
- Press `ENTER`. A window appears requesting information.
- Select `Data` if you have the sample data entered into a list.
- Otherwise, select `Stats`.

Hypothesis Testing on the TI-83

The `Stats` Option

TI-83 Hypothesis Testing for the Mean (`Stats` Option

- Enter μ_0, the hypothetical mean.
- Enter σ. (Remember, σ is known.)
- Enter $\overline{x}$.
- Enter n, the sample size.
- Select the type of alternative hypothesis.
- Select `Calculate` and press `ENTER`.

TI-83 Hypothesis Testing for the Mean

- A window appears with the following information.
  - The title `Z-Test`.
  - The alternative hypothesis.
  - The value of the test statistic Z.
  - The p-value of the test.
  - The sample mean.
  - The sample size.

Outline

Assignment

Homework

- Read Sections 10.1 - 10.2, pages 613 - 620.
- Let's Do It! 10.1, 10.2.
- Exercises 1 - 6, page 633.